

# No-boundary measure and preference for large $e$ -foldings in multi-field inflation

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## Abstract

The no-boundary wave function of quantum gravity tends to assign only very small probability to long periods of inflation. This leads to tension with observations. We study the no-boundary proposal in the context of multi-field inflation to see whether the number of fields changes the situation. For a simple model, we find that indeed the no-boundary wave function can give higher probability for sufficient inflation, but the number of fields involved has to be very high.

## 1 Introduction

The nature of the initial singularity of our universe is an important but unsolved problem in modern cosmology. One approach to this problem is to canonically quantize the universe and study its wave functions, solutions of the Wheeler-DeWitt equation [1]. There is no universally accepted way to

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choose, among the many solutions, the wave function that is supposed to describe our universe. One way to address this problem is to draw an analogy with the ground state in ordinary quantum systems. Hartle and Hawking [2] observed that the latter states can often be written in terms of the Euclidean path integral. Thus they suggested using an analogous definition in cosmology, resulting in what is called the no-boundary wave function.

The no-boundary wave function can be interpreted as a superposition of space-time histories. It can be approximated by sum-over on-shell solutions, so-called instantons. The sum-over instantons can have problems of divergence [3], which may be alleviated by considering complex valued instantons [4], so-called *fuzzy instantons* [5, 6].

As is to be expected in a quantum theory, the no-boundary wave function exhibits classical behavior only for a part of the phase space, i.e. only for certain space-time histories. Since we find ourselves in a universe which behaves classical on large scales, it is interesting to consider those histories in detail. It turns out that for this preferred subset of histories, the no-boundary wave-function assigns, to some approximation, a *classical* probability distribution, called the no-boundary measure. Recently, Hartle, Hawking and Hertog investigated the no-boundary measure for slow-rolling fields in Einstein gravity [5][6]; the present authors studied the no-boundary measure for more general examples: scalar-tensor gravity [7] and fast-rolling fields [8].

Due to previous studies, it was known that the no-boundary measure disfavors the histories which have a large number of  $e$ -foldings. If we believe that the early universe had experienced more than 60  $e$ -foldings from primordial inflation, the no-boundary measure is not compatible with this expectation.

On these grounds, one may reject the no-boundary wave function in favor of other prescriptions, for example Vilenkin's tunneling proposal [9], which may however be regarded as less natural on theoretical grounds. One may also try to keep the no-boundary wave function, but

- try to avoid the need for a long period of inflation such as ekpyrotic universe [10], string gas cosmology [11], or big bounce model [12],
- justify inflation on anthropic grounds, for example in the context of the multiverse [13],
- lift up the inflationary probabilities by considering the volume weighting [14][15].

These proposal each come with their own questions and difficulties attached. Therefore in this article we explore another mechanism to lift up the probability for the larger  $e$ -foldings in the context of the no-boundary measure. We study the no-boundary measure of *multiple fields* and argue that *the degeneracy of the large number of fields can lift up the probability of inflationary histories* without considering any additional factors. Although the Euclidean action does not prefer

inflationary histories, there exist many numbers of histories which produce larger  $e$ -foldings, if we include a number of fields. Then, for large numbers of fields, larger  $e$ -foldings can be sufficiently preferred.

In Section 2, we briefly review the no-boundary wave function and explain the reason why it does not prefer inflationary histories. In Section 3, the no-boundary measure is extended to the multi-field case and N-flation models, and it is demonstrated that the degeneracy can lift up the probability of larger  $e$ -foldings. We also relate the model with the observations. Finally, we summarize the results and discuss possible extensions in Section 4.

## 2 No-boundary wave function

The ground state of the universe, or the no-boundary condition of the wave function of the universe, can be described by the Euclidean path integral [2]:

$$\Psi[h_{\mu\nu}, \chi] = \sum_{\mathcal{T}} \int \mathcal{D}g_{\mu\nu} \mathcal{D}\phi e^{-S_E[g_{\mu\nu}, \phi]}, \quad (1)$$

where  $\mathcal{T}$  denote compact manifold topologies with a single spherical boundary, and the metric  $g_{\mu\nu}$  and the matter field  $\phi$  take the value  $h_{\mu\nu}$  and  $\chi$  at the boundary. Here,  $S_E$  is the Euclidean action: we assume Einstein gravity and a minimally coupled scalar field,

$$S_E = - \int d^4x \sqrt{+g} \left( \frac{1}{16\pi} R - \frac{1}{2} (\nabla\phi)^2 - V(\phi) \right) \quad (2)$$

in the units  $\hbar = G = c = 1$ . In this paper, for simplicity, we restrict our interests to a minisuperspace; for a homogeneous and isotropic universe, the metric becomes

$$ds^2 = N(\lambda)^2 d\lambda^2 + a(\lambda)^2 d\Omega_3^2. \quad (3)$$

In this context, it is convenient to view the time integral in the action as a contour integral in the complex time plane (for details see for example [5]) and shift the contour in suitable ways. Thus,  $\lambda$  (which we take to be in  $[0, 1]$ ) is a parametrization of the chosen contour. The no-boundary wave function takes the form

$$\Psi[b, \chi] = \int \mathcal{D}N \mathcal{D}a \mathcal{D}\phi e^{-S_E[a, \phi]}, \quad (4)$$

where  $a(\lambda = 1) = b$  and  $\phi(\lambda = 1) = \chi$  are the boundary values at the only boundary of space-time, and at  $\lambda = 0$  one imposes conditions suitable for a regular metric (“no-boundary conditions”).

The path integral can be approximated by the method of steepest descent. To lowest order, in this approximation we only keep the contribution from saddle points of the exponent:

$$\Psi[b, \chi] \simeq \sum_p e^{-S_E^p[b, \chi]} \quad (5)$$

where  $S_E^p[b, \chi]$  is the action for the solution labeled by  $p$  with the boundary conditions  $b, \chi$ , of the equations

$$\ddot{\phi} = -3\frac{\dot{a}}{a}\dot{\phi} + \frac{\partial V(\phi)}{\partial \phi}, \quad \ddot{a} = -\frac{8\pi}{3}a(\dot{\phi}^2 + V(\phi)), \quad (6)$$

where  $\dot{\phantom{x}}$  denotes derivative with respect to  $\tau$  defined by

$$\tau(\lambda) \equiv \int^\lambda d\lambda' N(\lambda'). \quad (7)$$

The on-shell action can be simplified to

$$S_E = 4\pi^2 \int d\tau \left( a^3 V - \frac{3}{8\pi} a \right). \quad (8)$$

We are especially interested in regions in which the wave function behaves almost classically. In the saddle point approximation (5), this translates to the condition

$$|\nabla S_E^{\text{Re}}[b, \chi]| \ll |\nabla S_E^{\text{Im}}[b, \chi]|, \quad (9)$$

which singles out Euclidean saddle points that contribute in the classical region.  $S_E^{\text{Re}}$  and  $S_E^{\text{Im}}$  are the real and imaginary parts of the Euclidean action and  $\nabla$  is a derivation that is defined on the superspace. It can be shown [5] that in this region the wave function effectively assigns classical probabilities for parts of the phase space, i.e. for some space-time histories. In fact, for a choice of contour such that the last part runs along the Lorentzian time axis, the histories that are assigned probability are *precisely* the histories satisfying (9), when evolved further in Lorentzian time. Hence we will call such histories *classical histories*. The probability measure for classical histories becomes

$$dP[b, \chi] \propto |\Psi[b, \chi]|^2 db d\chi \simeq e^{-2S_E^{\text{Re}}[b, \chi]} db d\chi. \quad (10)$$

The no-boundary measure is defined on the boundary of the histories,  $b$  and  $\chi$ . However, it is convenient to label a history not by the boundary values, but by the initial conditions, for slow-roll cases, since the Euclidean action is mostly determined by the initial potential energy:

$$S_E \simeq -\frac{3}{16} \frac{1}{V(\varphi)}, \quad (11)$$

where  $\varphi \equiv \phi(\tau = 0)$ . In these cases, one thus has

$$dP[\varphi] \propto \exp\left(\frac{3}{8} \frac{1}{V(\varphi)}\right) d\varphi. \quad (12)$$

as an approximation of the probability to observe a history starting from  $\varphi$  in the Euclidean regime.

We take the simplest inflation model for single field as one having a quadratic potential [17]

$$V(\phi) = \frac{1}{2} m^2 \phi^2. \quad (13)$$

It is known that except for  $\varphi = 0$ , only  $\varphi$  larger than a critical value has the solution space satisfying Equation (9) [6]. That is, the classical history exists only for

$$\varphi > \varphi^c \sim 0.62. \quad (14)$$

Although the no-boundary measure diverges at  $\varphi = 0$ , we will ignore this point. Due to the divergence it is impossible to evaluate the derivatives in (9), and it is hard to see how this history could be classical.

In the slow-roll approximation [18], the total  $e$ -folding number obtained from  $\varphi$  is approximated by

$$N_e \simeq -8\pi \int_{\varphi} d\phi \frac{V(\phi)}{V(\phi)_{,\phi}}, \quad (15)$$

where we have ignored a small correction from the location of the end of inflation. This can be further reduced for the quadratic potential Equation (13):

$$N_e \simeq 2\pi\varphi^2. \quad (16)$$

Then, the no-boundary measure, Equation (12), can be expressed as a function of  $e$ -folding number<sup>1</sup>

$$dP[N_e] \propto \exp\left(\frac{3\pi}{2} \frac{1}{m^2 N_e}\right) dN_e. \quad (17)$$

Here, the Jacobian of the change of variables is ignored because the exponential part is dominant. The classical histories exist for

$$N_e > N_e^c \equiv 2\pi(\varphi^c)^2 \sim 2.4. \quad (18)$$

Therefore, the no-boundary measure exponentially prefers a small  $e$ -folding number ( $\sim 2.4$ ).

As discussed in Section 1, there are several possible interpretations of this disagreement between the prediction of the no-boundary measure and the inflationary universe. One of the idea was that the volume factor should be weighted to measure the probability to observe our past light cone. It enhances the likelihood of larger  $e$ -foldings by multiplying the volume [5],

$$dP[N_e] \propto \exp\left(\frac{3\pi}{2} \frac{1}{m^2 N_e} + 3N_e\right) dN_e. \quad (19)$$

This volume weighting sufficiently lifts up the probability of larger  $e$ -foldings, if

$$N_e \gtrsim m^{-2}. \quad (20)$$

In the following, we will study under which conditions the addition of further fields may have a similar effect.

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<sup>1</sup>Since the expansion during the Euclidean time is negligible, Equation (16) is still a good approximation.

### 3 No-boundary measure in multi-field inflation

In this section, we study the no-boundary measure with multiple fields. As simple toy model, we take all fields, having same quadratic potential type for simplicity as in the earlier works on N-flation models [19, 20]<sup>2</sup> where it was shown that any of these fields does not need to take on values in excess of the Planck scale<sup>3</sup> and a large number of fields, predicted by string vacuum solutions, is needed to get sufficient  $e$ -foldings. We take N-flation models as a possible candidate of the multi-field inflation models throughout this paper. The basic effect that we will be investigating is that, although large amounts of inflation are still suppressed in terms of the action, there are now many histories that contribute to a given amount of inflation. This volume in field space competes with the suppression due to the action.

#### 3.1 No-boundary wave function of multiple fields

The no-boundary wave function in Equation (1) can be extended to the multi-field case. In the minisuperspace of scale factor and a set of scalar fields, it becomes

$$\Psi[b, \vec{\chi}] = \int \mathcal{D}N \mathcal{D}a \mathcal{D}\vec{\phi} e^{-S_E[a, \vec{\phi}]} \quad (21)$$

where the arrow denotes multiple fields,  $\vec{\phi} \equiv \{\phi_j\}$ , and  $a(\lambda = 1) = b$  and  $\vec{\phi}(\lambda = 1) = \vec{\chi}$  and the parametrization  $\lambda$  was chosen such that  $\lambda = 1$  corresponds to the single boundary.

As discussed in Section 2, we use the method of the steepest descent

$$\Psi[b, \vec{\chi}] \simeq \sum_p e^{-S_E^p[b, \vec{\chi}]}, \quad (22)$$

where

$$\ddot{\phi}_i = -3\frac{\dot{a}}{a}\dot{\phi}_i + \frac{\partial V(\vec{\phi})}{\partial \phi_i}, \quad \ddot{a} = -\frac{8\pi}{3}a\left(\dot{\vec{\phi}}^2 + V(\vec{\phi})\right), \quad (23)$$

$$S_E = 4\pi^2 \int d\tau \left( a^3 V(\vec{\phi}) - \frac{3}{8\pi} \dot{a}^2 \right), \quad (24)$$

where the subscript  $i$  denotes the  $i$ -th field.

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<sup>2</sup>Earlier investigations of the N-flation paradigm [19, 20, 21, 22] have worked under the assumption that all relevant fields are close to their minima and can be described by quadratic potentials. For axions the full potential is trigonometric and recently Refs. [23] showed that the quadratic approximation is unreliable. The reason we follow the earlier works rather than the more recent ones is mainly for simplicity. We intend to do further work on for the case of more complicated potentials.

<sup>3</sup>Note that, in literature, people use *reduced* Planck mass scale.

### 3.2 Degeneracy of solution space for the case of a quadratic potential

We consider a set of uncoupled fields in quadratic potentials with the same mass  $m_i = m$  [19, 20],

$$V(\vec{\phi}) = \frac{1}{2}m^2\vec{\phi}^2. \quad (25)$$

The equations of motion are

$$\ddot{\vec{\phi}} = -3\frac{\dot{a}}{a}\dot{\vec{\phi}} + m^2\vec{\phi}, \quad \ddot{a} = -\frac{8\pi}{3}a\left(\dot{\vec{\phi}}^2 + \frac{1}{2}m^2\vec{\phi}^2\right). \quad (26)$$

The no-boundary conditions at  $\tau = 0$  require  $\dot{\vec{\phi}} = 0$ . Thus we can make the ansatz

$$\vec{\phi}(\tau) = \phi(\tau)\hat{\phi}, \quad (\hat{\phi})^2 = 1, \quad (27)$$

which requires  $\dot{\vec{\phi}} = (\dot{\phi}/\phi)\vec{\phi}$ . Then the equations of motion for  $\phi(\tau)$  and  $a(\tau)$  are precisely given by the equations (6) for a single scalar in a quadratic potential. The no-boundary wave function is thus independent of the direction of  $\vec{\phi}$  in field space,

$$\Psi(b, \vec{\chi}) \equiv \Psi(b, \chi), \quad \chi = |\vec{\phi}|. \quad (28)$$

As a result, the classicality conditions in this case reduce to those of the single field case, and one can use Equations (12) and (14). Similar to Equation (12), the no-boundary measure can be defined on the initial field space in the slow-roll limit,

$$dP[\vec{\varphi}] \propto \exp\left(\frac{3}{4}\frac{1}{m^2\varphi^2}\right) d\vec{\varphi} \quad (29)$$

and we have again ignored a Jacobian due to the change of variables.  $\varphi \equiv |\vec{\varphi}|$ . Important physical observables, e.g., the  $e$ -folding number and power spectrum, do not depend on the angular part of  $\vec{\varphi}$ . Therefore, for these observables the effective probability distribution is

$$dP[\varphi] = \int d\hat{\varphi} dP(\hat{\varphi}, \varphi) \simeq D(\varphi^2) \exp\left(\frac{3}{4}\frac{1}{m^2\varphi^2}\right) d\varphi \quad (30)$$

where  $|\hat{\varphi}| = 1$  and  $D(\varphi^2)$  is the surface area of the shell  $|\vec{\varphi}| = \varphi$  in field space.

Now we need to discuss the issue of cutoffs in (30): On the one hand, there are no classical solutions below a certain threshold, see (18). On the other hand, the probability distribution (30) can not be normalized since it becomes constant for large values of  $\varphi$ , and a cutoff is needed. Here, we assume that all fields are below  $\alpha$  that is order of the Planck mass  $\sim 1$ . This is indeed the philosophy of the N-flation scenario [19, 20, 21, 23]. If a field value is larger than the Planck scale, it is very difficult to trust their self-consistency of the effective theory; therefore, it is reasonable to restrict the field values less than the Planck scale and consider assisted inflation. Furthermore, if one assumes the axion fields, the cutoff arises naturally.

We will account for the cutoffs by changing  $D(\varphi^2)$  appropriately. To estimate  $D(\varphi^2)$ , we choose  $\varphi_i$  randomly and count resulting  $\varphi^2$ . It can be done by assuming  $\varphi_i$  as a uniform random variable and using the central limit theorem. We assume that there are  $N_f$  number of fields with initial field value  $\varphi_i$  following

$$\varphi_i \in \mathcal{U}(0, \alpha), \quad (31)$$

where  $\mathcal{U}(p, q)$  is the uniform distribution with minimum  $p$  and maximum  $q$ . For the uniform distribution of  $\varphi_i$ , the mean and the variance of  $\varphi_i^2$  become

$$E[\varphi_i^2] = \frac{1}{\alpha} \int_0^\alpha d\varphi_i \varphi_i^2 = \frac{\alpha^2}{3}, \quad (32)$$

$$\sigma^2[\varphi_i^2] = E[\varphi_i^4] - E[\varphi_i^2]^2 = \frac{4\alpha^4}{45}. \quad (33)$$

When  $N_f$  is large, a statistical behavior will arise, e.g., it will be more probable to observe  $\varphi^2$  around its expectation value. It is described by the central limit theorem which states that the distribution of  $E[\varphi^2]$  follows the Gaussian distribution  $\mathcal{N}(\mu, \sigma^2)$  with the mean  $\mu$  and the variance  $\sigma^2$ , such that

$$E[\varphi^2] \in \mathcal{N}(\mu, \sigma^2), \quad (34)$$

$$\mu \equiv \frac{\alpha^2 N_f}{3}, \quad (35)$$

$$\sigma^2 \equiv \frac{4\alpha^4 N_f}{45}. \quad (36)$$

This distribution corresponds to the relative number of equivalent choices of  $\vec{\varphi}$  for a given  $\varphi^2$ . Therefore,

$$D(\varphi^2) \propto \exp\left(-\frac{(\varphi^2 - \mu)^2}{2\sigma^2}\right). \quad (37)$$

Then, the no-boundary measure of  $\varphi^2$  follows from Equation (30):

$$P[\varphi^2] \propto \exp\left(\frac{3}{4} \frac{1}{m^2 \varphi^2} - \frac{(\varphi^2 - \mu)^2}{2\sigma^2}\right). \quad (38)$$

It represents the Euclidean probability measure and the degeneracy of the configuration space at the same time.

Now we turn our attention to the  $e$ -folding number. The degeneracy factor can be described in terms of the  $e$ -folding number by using Equation (16),

$$E[N_e] \in \mathcal{N}(\tilde{\mu}, \tilde{\sigma}^2), \quad (39)$$

$$\tilde{\mu} \equiv \frac{2\pi\alpha^2 N_f}{3}, \quad (40)$$



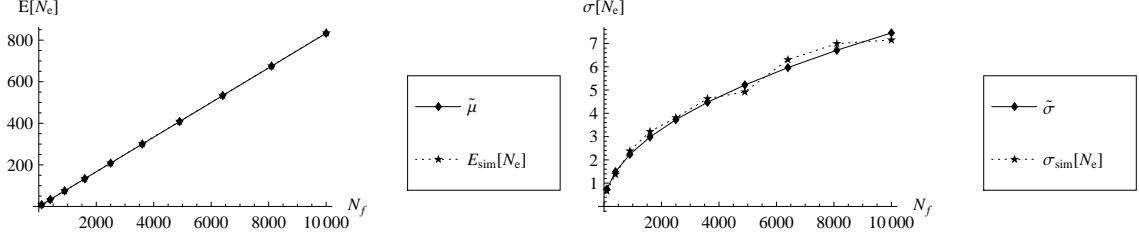


Figure 1: Comparison between  $\tilde{\mu}$  and  $E_{\text{sim}}[N_e]$  and between  $\tilde{\sigma}^2$  and  $\sigma_{\text{sim}}^2[N_e]$ . Subscript sim denotes the numerical values obtained from 100 simulations for each  $N_f$ . We used the uniform random  $\varphi_i$  following Equation (31) with  $\alpha = 1/\sqrt{8\pi}$  which corresponds to the initial field value below the reduced Planck mass. Only Lorentzian evolution is considered in these simulations, since the expansion during the Euclidean time is negligible.

$$\tilde{\sigma}^2 \equiv \frac{16\pi^2\alpha^4 N_f}{45}. \quad (41)$$

This distribution is confirmed by the simulation in Figure 1. The no-boundary measure also can be denoted by the  $e$ -folding number:

$$\begin{aligned} dP[N_e] &\propto \exp\left(\frac{3\pi}{2} \frac{1}{m^2 N_e} - \frac{(N_e - \tilde{\mu})^2}{2\tilde{\sigma}^2}\right) dN_e \\ &= \exp\left(\frac{3\pi}{2} \frac{1}{m^2 N_e} - \frac{5}{8} N_f \left(1 - \frac{N_e}{\tilde{\mu}}\right)^2\right) dN_e. \end{aligned} \quad (42)$$

The degeneracy factor enhances the probability at  $N_e = \tilde{\mu}$  and reduces the probability at  $N_e = N_e^c$ . Therefore, it increases the likelihood of the larger  $e$ -folding number. This effect becomes stronger as  $N_f$  increases and overcomes the probability peak at  $N_e^c$  when

$$N_f \gtrsim m^{-2}, \quad (43)$$

since the parenthesis in the exponential of Equation (42) is less than one for  $N_e < \tilde{\mu}$ . Note that this condition is comparable to that of the volume factor in Equation (20), because when the cutoff  $\alpha$  is order of the Planck scale, a set of fields satisfying Equation (43) will produce enough  $e$ -folding number by satisfying Equation (20). Figure 2 summarizes the main result of this paper.

### 3.3 Cosmological viability

As previously mentioned, we take N-flation models in the same mass case [19, 20] as possible candidates of multi-field inflation and apply the results of the no-boundary measure.

Using the slow-roll approximation and for quadratic potentials, the number of  $e$ -foldings Eq.(15)

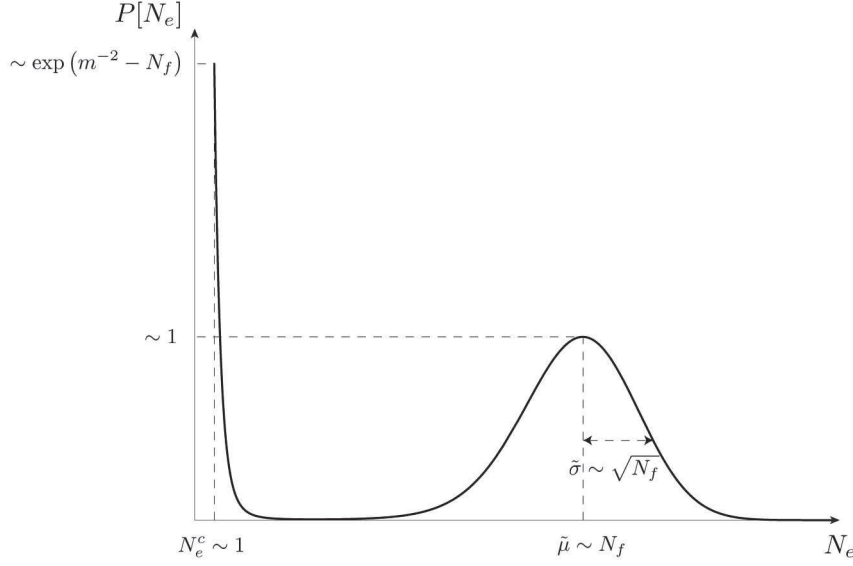


Figure 2: Schematic diagram of the no-boundary measure in Equation (42). For initial fields below the Planck scale, the degeneracy factor of the  $e$ -folding number has expectation value of the order of  $N_f$  while its standard deviation is order  $\sqrt{N_f}$ . When  $N_f \gtrsim m^{-2}$ , the peak at  $N_e^c$  which comes from the Euclidean probability measure becomes comparable to the peak at  $\tilde{\mu}$  which comes from the degeneracy of the field space. Therefore, the degeneracy factor enhances the likelihood of the larger  $e$ -folding number. Note that the probability in vertical axis is not normalized and intended to represent the relative probability.

can be rewritten as

$$N_e \simeq 2\pi \sum \phi_i^2. \quad (44)$$

We will assume throughout that the observable scales crossed outside the horizon 60  $e$ -foldings before the end of inflation  $N_e^* \sim 60$  [26].

The power spectrum, the spectral index, the tensor-to-scalar ratio, and the non-gaussianity parameter<sup>4</sup> are given by [20]

$$P_{\mathcal{R}} \simeq \frac{128\pi}{3} V(\{\phi_j^*\}) \sum_{i=1}^{N_f} \left( \frac{V(\phi_i^*)}{V(\phi_i^*, \phi_i)} \right)^2 \simeq \frac{16\pi}{3} m^2 \left( \sum_i (\phi_i^*)^2 \right)^2, \quad (45)$$

$$n_S - 1 \simeq -\frac{1}{\pi \sum_i \phi_i^2} \simeq -\frac{2}{N_e^*}, \quad (46)$$

$$r \simeq \frac{4}{\pi \sum_i \phi_i^2} \simeq \frac{8}{N_e^*}, \quad (47)$$

$$\frac{6}{5} f_{\text{NL}} \simeq \frac{1}{4\pi \sum_i \phi_i^2} \simeq \frac{1}{2N_e^*}. \quad (48)$$

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<sup>4</sup>Here the non-gaussianity is local type.

For  $N_e^* \sim 60$  [26], these becomes are given by

$$n_S - 1 \sim 0.033, \quad (49)$$

$$r \sim 0.133, \quad (50)$$

$$\frac{6}{5} f_{\text{NL}} \sim O(0.01). \quad (51)$$

Even though this case does not give significantly large non-gaussianity, but still all these parameters are still within viable region of WMAP7 [25].

We can reduce the formula for the power spectrum in terms of effect single field as

$$P_{\mathcal{R}} \simeq \frac{16\pi}{3} m^2 (\phi^*)^4. \quad (52)$$

The observed normalization of the spectrum can be taken as  $P_{\mathcal{R}}^{1/2} \simeq 5 \times 10^{-5}$  [25], leading to a normalization of

$$m \sim 1.3 \times 10^{-6}, \quad (53)$$

which is a reasonable mass scale for the quadratic model regardless of the choice of  $N_f$ . Therefore, from Equation (43), the multi-field factor works for  $N_f \gtrsim 10^{12}$  which is quite large number of fields needed.

## 4 Discussions and conclusion

In this article, we investigated the no-boundary measure with  $N_f$  fields. To obtain a very simple model, we assumed a quadratic potential with a same mass  $m$ . Following the spirit of N-flation, we further assumed that the field values of each fields should be less than the Planck scale:  $\alpha \lesssim 1$ . Then, the no-boundary measure effectively has two competing factors (Equation (38)): the Euclidean action and the Gaussian degeneracy factor.

If the number of fields  $N_f$  becomes sufficiently large so that  $N_f \sim m^{-2}$ , then the two factors are of similar orders. The peak of the Gaussian factor  $\varphi \sim N_f$  then gives realistic cosmologies non/negligible probability. However, the nuber of fields needed to achieve this effect seems to be very large. In the simple model we considered,  $N_f \sim 10^{12}$  number of fields are needed to fit the power spectrum of our universe. Since this is a very high number, this seems to suggest that this is not an effective way to resolve the tension between no-boundary probabilities and the observations. But there are also less pessimistic ways to view the situation:

1. A large number of scalar fields is not ruled out by experiments if their energy scale is near Planck scale. Indeed it has been argued that such a scenario is useful to resolve some problems of our phenomenological universe [16]. Therefore, it may still be a viable idea.

2. In our calculation, we assumed a *quadratic potential* described by a *single constant mass*, but one can certainly question these assumptions.

- (a) The masses can depend on the energy scale or the time, via the running of couplings (masses) or the temperature dependence. If the effective mass is sufficiently large when the universe begins inflation, and if it decreases as time goes on, then one may explain the sufficient  $e$ -foldings with reasonable numbers of fields.
- (b) The mass can be different for different fields, e.g. Refs.[20, 22]. If the Euclidean dynamics highly depends on the larger mass, then the required number of fields to lift up the larger  $e$ -foldings can be reduced. This can explain the power spectrum, if the smallest mass is order of  $10^{-6}$ .
- (c) The quadratic potential should only be regarded as a first approximation of more complicated potentials(e.g., axions in Refs. [23]). More complicated potentials may change the Euclidean cutoffs and reduce the peak of the Euclidean probability, leading to a required  $N_f$  more in keeping with present models of high energy physics.
- (d) If the fields couple in complicated ways, then again, the Euclidean cutoff structures can be changed. Therefore, from the similar arguments, one may reduce the number of fields to a reasonable amount.

To summarize, we have shown that considering the no-boundary wave function for many fields can solve the traditional problem of low probabilities for sufficient inflation. But in the simple model we have considered, the number of fields is very high, much higher than supported by, for example, string theory scenarios. Further research is needed to see whether the high number of necessary fields can be brought down in more complicated and realistic models.

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of Laila Alabidi, Takahiro Tanaka and YITP, and of CQeST while this work was being carried out.

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